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SOME AERODYNAMIC STUDIES OF RE-ENTRY CONFIGURATIONS IN BALLISTIC RANGES

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ABSTRACT

The problems of extracting stability data from the measured attitude and positional coordinates of models fired in a ballistic range are discussed and representative experimental data is presented. A significant problem occurs when large nonlinear forces and moments are present and also when there is strong intercoupling between the lateral and longitudinal modes. Complete solutions for such motions have not been found; however, some special cases may be solved.

INTRODUCTION

The investigation of the performance of re-entry bodies which are representative of either the ballistic missile or the manned vehicle presents a considerable challenge to the aerodynamicist. Investigations must be performed over a wide range of Mach and Reynolds numbers and must include real gas phenomena such as dissociation and relaxation effects, ablation, and viscous interaction effects. Although no ground facility exists for full simulation of all the various phenomena, many laboratory facilities are under development to more closely simulate the flow conditions. The ballistic range is one such facility: it possesses the advantage of allowing the study of flow-fields about scaled models under free-flight conditions. Simultaneous measurements of model performance, surface conditions, emitted radiation, and microwave interaction with the ionized plasma may be obtained.

A significant problem in vehicle design involves the measurement of the stability parameters and their dependence on model geometry for input into re-entry trajectory analyses. Configurations designed to withstand the high thermal and deceleration loadings of re-entry often possess nonlinear stability parameters and exhibit severe intercoupling between the lateral and longitudinal motions. A primary function of the ranges of the Canadian Armament Research and Development Establishment (CARDE) is the exploration of these effects. The measurements of the model trajectory during its short flight in the range must be thoroughly examined to extract relatively small quantities, since small non-linearities or cross-coupling effects may not be too obvious over the short-range trajectory, whereas they may present serious problems in the long re-entry flight paths.

Most of the data presented in this report has been obtained at supersonic speeds. The methods of analyzing the range data are applicable to the hypersonic as well as the supersonic speed regimes.

RANGE TECHNIQUES

The ballistic range technique consists of launching scaled model configurations from guns into instrumented ranges. The basic measurements obtained are attitude, position, and model transit time. Photographs showing the model in flight and the associated flow-field patterns are also obtained. The attitude and position coordinates may be measured using orthogonal photographic stations or lightweight papers (yaw cards) mounted at regular intervals perpendicular to the flight path.[1] The data presented in this paper has been obtained by the latter method. The resultant trajectory histories may then be used to extract the stability parameters of the model.

The model-launching device may be either a conventional powder gun or a light-gas gun. The propellant gases from powder guns possess a high molecular weight, and therefore a considerable amount of the available energy is used to accelerate the propellant gases in the gun barrel. Recent developments in applying hydrogen and helium gases as the propellant have greatly increased the amount of energy transferred to the projectile by the driving gases. Projectile velocities of the order of 30,000 fps have been obtained with light gas guns.

With the availability of very high velocities in the ballistic range many new measuring techniques have been developed. From the viewpoint of aerodynamics a need exists for measurements of the flow-fields. Pressure, temperature, gas species and concentration and model surface conditions must be determined. Description of the CARDE facilities including instrumentation are available in Refs. 2, 3, and 4. Telemetering units designed to withstand the very high gun-launching accelerations have been successfully developed and currently many forms of transducers are being investigated.[5] Of particular interest to the aerodynamicist is the development of heat-transfer gages and accelerometers which are carried in the model. The information from them will be transmitted from the model in flight by means of telemetry. The merits of accelerometers for data extraction will be discussed later in this paper.

The ballistic ranges as operated at CARDE provide simultaneous measurements of attitude, velocity, in-flight photography, peak and selected bands of emitted radiation[3] and microwave interferometry[4] of the gas flow. In some cases several channels of telemetry may be added to the model to measure conditions internal to the model and/or on its surface. The standard techniques of flow-field investigation such as schlieren and interferometry (in the visible light region) are not particularly suited to the high-velocity, low-density studies. Two examples of in-flight photography are shown in Figs. 1 and 2. A schlieren photograph of a body of revolution traveling at approximately 6,000 fps in air at atmospheric pressure is shown in Fig. 1. Figure 2 shows the self-luminosity of a plastic sphere of 0.6-in. diameter traveling at 15,000 fps in air at 80 mm Hg.

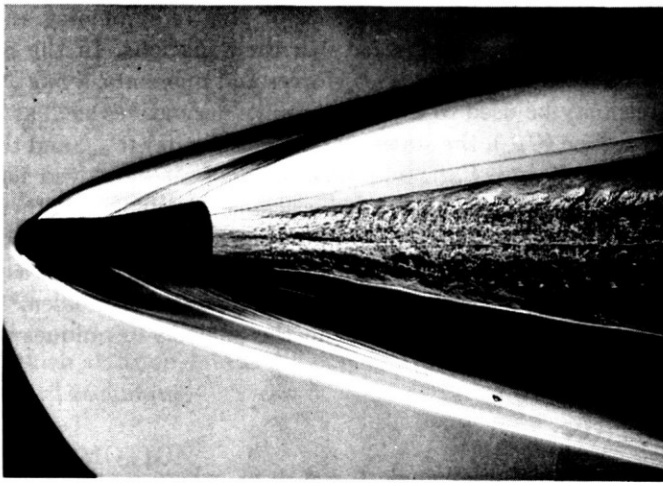

Fig. 1. Schlieren photograph of a body of revolution ($v \sim 6{,}000$ ft/sec).

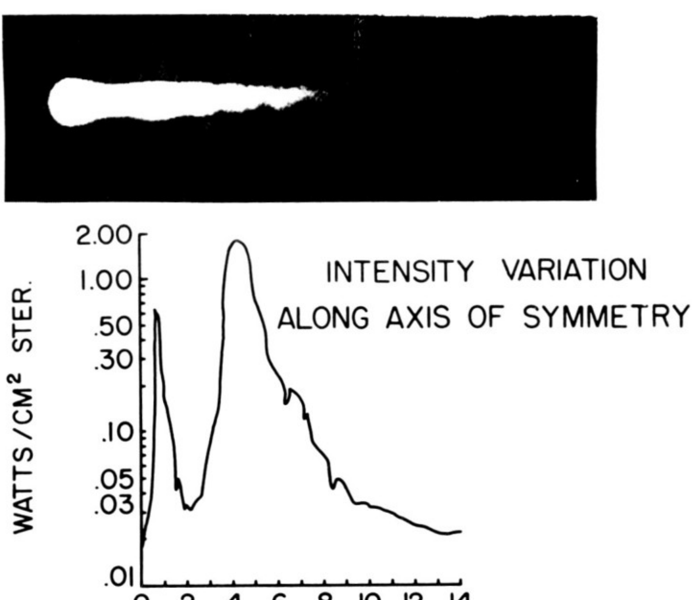

Fig. 2.

The photograph was obtained with an image converter camera with an exposure time of 0.4 μsec. The variation of intensity along the axis of symmetry of the body is also presented in Fig. 2. This was obtained by means of a microdensitometer reading of the photograph.

BODIES WITH TRIGONAL OR GREATER SYMMETRY

The majority of missiles possess trigonal or greater symmetry and they can exhibit varying degrees of non-linearity in their motions. In the case of configurations which have essentially linear forces and moments, a relatively simple form of analysis may be used to extract stability derivatives such as the normal force coefficient slope (C_{N_α}), the static stability parameter (C_{m_α}) and the damping moment coefficient ($C_{m_{\dot\alpha}} + C_{m_q}$). The form of analysis employed has been developed from the techniques of the ballisticians. Fowler and others[6] in 1920 demonstrated the epicyclic motion of spinning shells, and later, Nicolaides,[7] with the assumptions of constant roll rate and slight configurational asymmetries, produced a tricyclic analysis for the motion. Murphy,[8] Rasmussen,[9] Kirk[10] and others have devised methods which extend the analysis techniques to allow for small nonlinearities in the motion.

LINEAR MOTION

By introducing the complex angle $\xi = \beta + i\alpha$, where α and β are the angles of attack and sideslip in the rolling (or body) axis system, and by making various assumptions, the Euler equations of motion of a rigid body with trigonal or greater symmetry may be reduced to a single complex differential equation as follows:

$$\ddot{\xi} + A(t)\,\dot{\xi} + B(t)\xi = C(t) \tag{1}$$

$A(t)$, $B(t)$ and $C(t)$ are complex time-variable coefficients which depend on the aerodynamic stability characteristics and inertia properties of the body.

A transformation to non-rolling axes, $\hat{\xi} = \xi e^{ip_0 t}$, where p_0 is the steady roll rate, yields a solution of the form

$$\hat{\xi} = K_1 \exp\{(\lambda_1 + i\omega_1)t\} + K_2 \exp\{(\lambda_2 + i\omega_2)t\} + K_3 \exp\{ip_0 t\} \tag{2}$$

K_1, K_2, and K_3 are in general complex quantities with K_1 and K_2 being dependent on initial conditions; the frequencies and the damping factors ω_1 and ω_2, λ_1 and λ_2 are real and p_0 is the roll rate. The solution is found with the assumptions of linear forces and moments and constant flight velocity and roll rate and it is seen from Eq. (2) that the motion is tricyclic, i.e., it is defined by three rotating vectors in the $\hat{\alpha}$, $\hat{\beta}$ plane. Two of these vectors represent the damped oscillations of the model while the third vector represents the trim due to model asymmetry. Equation (2) in rolling coordinates becomes

$$\xi = \beta + i\alpha = K_1 \exp\{[\lambda_1 + i(\omega_1 - p_0)]t\} + K_2 \exp\{[\lambda_2 + i(\omega_2 - p_0)]t\} + K_3 \tag{3}$$

By subtracting each vector in turn from the motion it is usually quite straightforward to derive the frequencies (ω_1 and ω_2) and damping (λ_1 and λ_2) which lead directly to the stability derivatives. Descriptions of the method have been given by Bull,[1] Mantle,[11] and Nelson.[12]

Light oscillating models sometimes generate an adequate amount of lift to produce a measurable oscillation in the motion of the center of gravity. A solution for positional data similar to that given above for angular orientation may be derived as follows:

$$Y + iZ = \frac{1}{2}\frac{\rho S}{m} C_{N_\alpha} \left[\frac{K_1}{\omega_1{}^2} \exp\,(i\omega_1 X) + \frac{K_2}{\omega_2{}^2} \exp\,(i\omega_2 X) \right] \exp\left[\frac{\lambda_1 + \lambda_2}{2} X \right] \quad (4)$$

A combination of angular data, center of gravity coordinates and velocity can yield the following quantities from a single model firing: C_{N_α}, C_{m_α}, C_D, $C_{m_{\dot\alpha}} + C_{m_q}$ and center of pressure position. The additional use of center-of-gravity coordinates constitutes a definite improvement in that more experimental data is obtained and hence greater accuracy and confidence in the results may be achieved. It is also possible that data may be obtained from some symmetrical rounds for which attitude data is difficult or even impossible to obtain by means of the yaw-card technique.

NON-LINEAR MOTION

When small non-linearities are present in the aerodynamic moments a tricyclic solution may be obtained using a method developed by Murphy.[8] Here it is assumed that the motion is quasilinear over short trajectories and that the epicyclic vectors K_1 and K_2 are functions of some effective magnitude of the motion. The variation of the stability parameters with amplitude of motion may be obtained by firing several models of the same configuration at the same Mach number but over a range of angles of attack. In order to predict the motion of a missile over long portions of its trajectory, Murphy uses the amplitude plane in which the squares of the modal amplitudes ($K_1{}^2$ and $K_2{}^2$) are plotted one against the other. Each type of non-linearity (e.g., in static moment C_{m_α} or Magnus moment $C_{m_{p\alpha}}$) has its own characteristic amplitude plane diagram and once the type on non-linearity present and the initial conditions are known, it is possible to predict the motion which follows.

The analysis of the motion when large non-linearities are present is a difficult task. Some attempts which were made to analyze the highly non-linear motion of low-aspect-ratio delta wings are discussed below. An example of the motion of an axisymmetric body where large non-linearities appear to be present is shown in Fig. 3*a*. The model was a hemisphere-cylinder-flare configuration and Fig. 3*a* shows the angular variations with distance downrange in rolling coordinates (α, β). The unsymmetrical character of the oscillations is quite noticeable and a standard tricyclic analysis did not produce the constant values of damping and frequency which are required when a quasilinear analysis of the Murphy type is to be used. The preliminary analysis of the behavior of the individual vectors has shown some interesting features. On plotting the "epicyclic" vectors on the amplitude plane the curve appears to oscillate and approach a limit line as shown in Fig. 3*b*. The explanation for this behavior is not fully understood but the investigations are being continued as part of a more general study of the effects of large non-linearities on the motion of bodies with trigonal or greater symmetry.

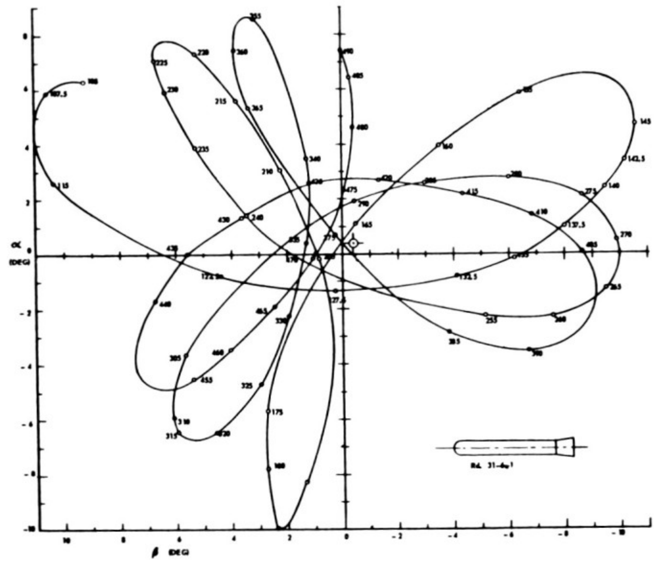

Fig. 3*a*. Attitude history (rolling coordinates).

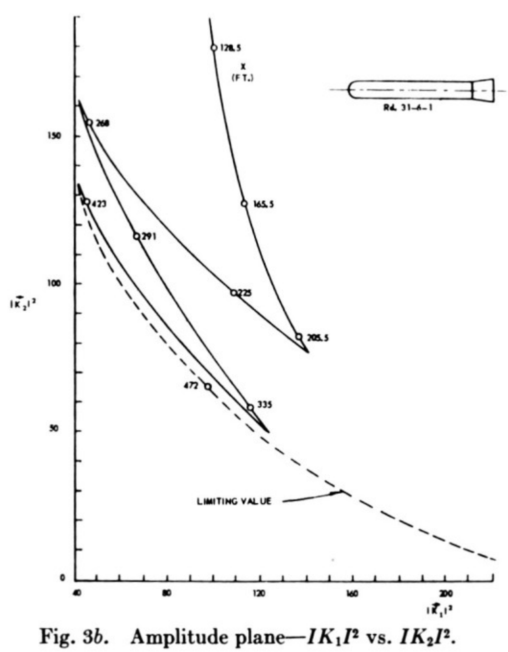

Fig. 3*b*. Amplitude plane—IK_1I^2 vs. IK_2I^2.

LIFTING CONFIGURATIONS

Aircraft and manned lifting re-entry vehicles do not possess the necessary symmetry to allow the analysis of their free-flight motion by the tricyclic method.

The trajectory data obtained from a test of an airplane model is shown in Fig. 4. The longitudinal motion consists of a simple uncoupled pitching oscillation which may be solved with a one-degree-of-freedom equation. The lateral motion is seen to be typical of the Dutch roll motion. The roll and yaw equations used to define this motion are as follows:

$$\ddot{\phi} - \frac{I_{xz}}{I_x}\ddot{\psi} = \frac{\rho V^2 Sb}{I_x}\left\{C_{1_\beta}\beta + C_{1_p}\frac{pb}{2V} + C_{1_r}\frac{rb}{2V}\right\}$$
$$\ddot{\psi} - \frac{I_{xz}}{I_z}\ddot{\phi} = \frac{\rho V^2 Sb}{I_z}\left\{C_{n_\beta}\beta + C_{n_p}\frac{pb}{2V} + C_{n_r}\frac{rb}{2V}\right\} \tag{5}$$

where it is assumed that side forces are negligible (i.e., $\psi = -\beta$). On the basis of the character of the records, solutions of the following form were assumed:

$$\psi = \psi_0 e^{-Kt}\sin\omega t$$
$$\phi = \phi_0 e^{-K^t}\sin(\omega t + \theta) \tag{6}$$

By taking values of 0 and $\pi/2$ for ωt it is then possible to obtain four algebraic equations from Eq. (5). As there are six unknown derivatives to be solved from

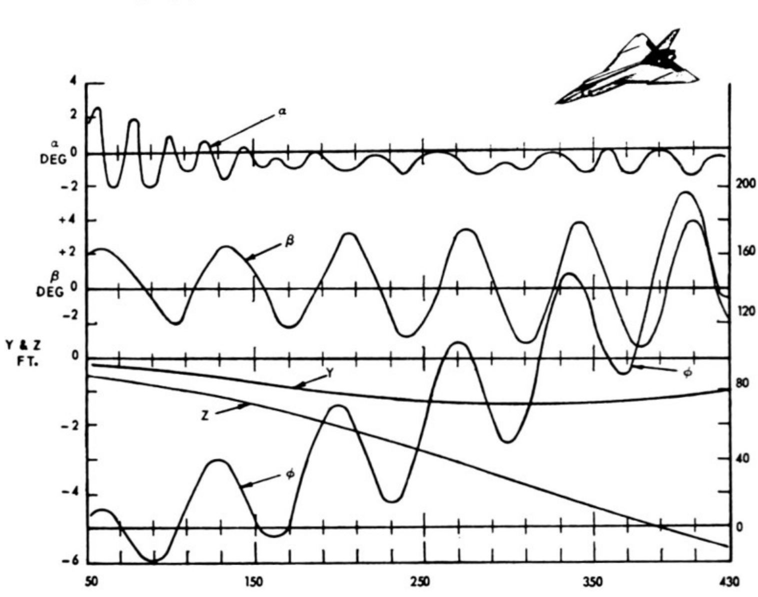

Fig. 4. Trajectory history.

the four equations, it is necessary to assume values for two and solve for the remaining four unknowns. The choice of the two values to be assumed depends on the characteristics of the particular configuration and also on what information is available from other sources (wind tunnels, for example).

For measurements of wavelengths (λ) and damping (K_α) the static and dynamic pitching moment derivatives may be derived from the longitudinal motion by means of the following expressions for the static stability parameter

$$C_{m_{\dot{\alpha}}} = -\frac{2}{\rho}\frac{I_y}{Sc}\left(\frac{2\pi}{\lambda}\right)^2 \tag{7}$$

and the damping in pitch

$$C_{m_{\dot{\alpha}}} + C_{m_q} = \frac{2I_y}{c^2}\left\{\frac{C_{N\alpha}}{m} - \frac{4K_\alpha}{\rho VS}\right\} \tag{8}$$

From values of C_{m_α} corresponding to different model center-of-gravity positions it is possible to calculate C_{N_α} and the center-of-pressure position.

The lateral and longitudinal derivatives obtained for the delta-winged airplane configurations obtained in the ballistic range are compared with wind-tunnel results in Figs. *5a* and *5b*. The wind-tunnel data were obtained at the

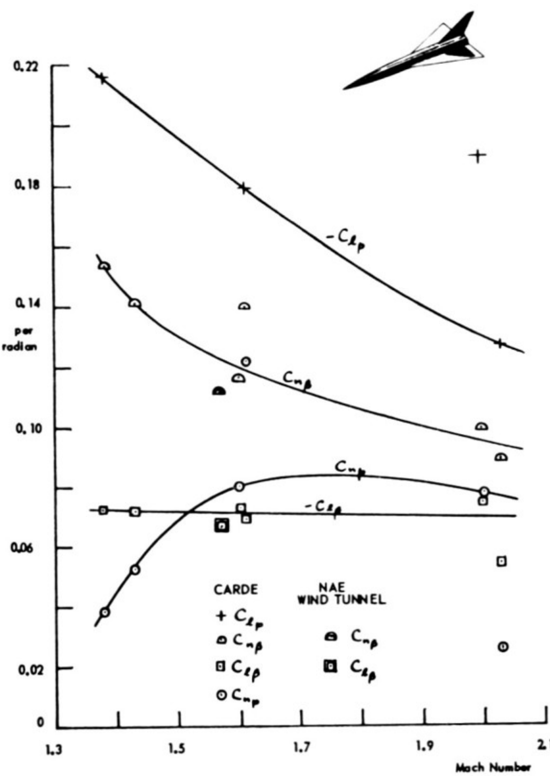

Fig. *5a*. Lateral derivatives.

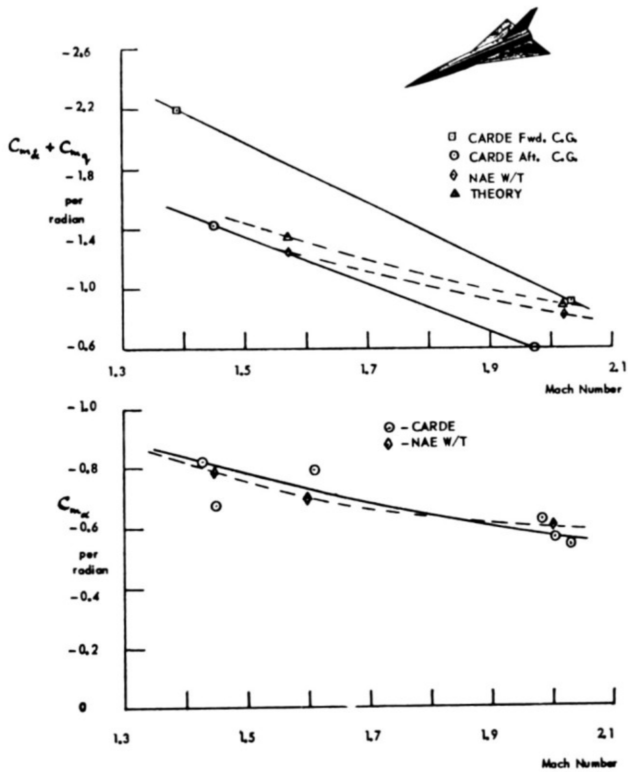

Fig. 5b. Longitudinal derivatives.

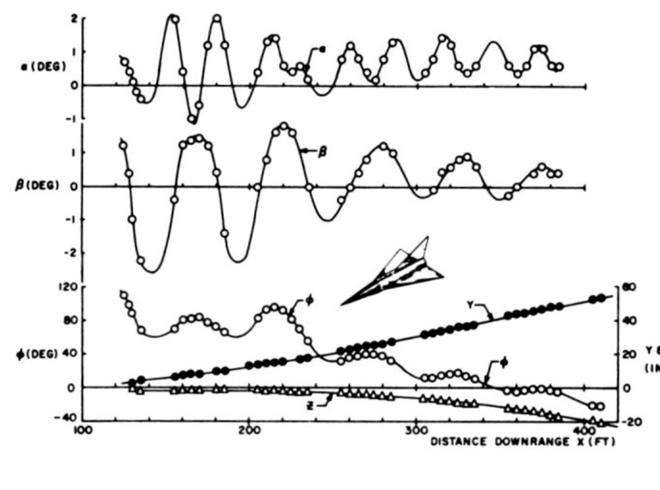

Fig. 6. Hypersonic glider trajectory.

National Aeronautical Establishment (Canadian) supersonic tunnel using the half-model techniques.

The above analysis methods are applicable for winged configurations provided that the longitudinal and lateral motions are uncoupled. In the case of a delta-winged hypersonic glider, it can be seen from the trajectory data in Fig. 6 that some cross-coupling is present, thus the need for a more complex form of analysis is indicated. The following section on the flat-plate delta-wing study describes the methods with which CARDE has attempted to solve the equations of motion where the intercoupling and large non-linearities are influencing the motion.

FLAT-PLATE DELTA-WING STUDY

The delta wing has many applications ranging from supersonic transports to vehicles used for transfer from an orbital flight to re-entry into the atmosphere. Some of the studies conducted at CARDE on this configuration are summarized here. The wings under study varied in aspect ratio ($AR = \frac{1}{2}$, 1 and 2).

ANALOG SIMULATION STUDIES

The equations of motion which are adequate to describe the wing's trajectory are determined by matching the experimental values of pitch, yaw and roll with those predicted from the theoretical stability derivatives. The equations selected which appear to give the best trajectory representation are as follows:

$$\dot{\beta} - p\alpha + r = \frac{Y_\beta}{mU}\beta + \frac{g}{U}\sin\phi$$

$$\dot{\alpha} - q + p\beta = \frac{Z_\alpha}{mU}\alpha + \frac{Z_{\alpha 2}}{mU}\cdot\alpha^2$$

$$\dot{p} - \left\{\frac{I_y - I_z}{I_x} + \frac{L_{qr}}{I_x}\right\}qr = \frac{L_{\alpha\beta}}{I_x}\alpha\beta + \frac{L_p}{I_x}p + \frac{L_r}{I_x}r + \frac{L_{\alpha r}}{I_x}\alpha r \qquad (9)$$

$$\dot{q} - \frac{I_z - I_x}{I_y}pr = \frac{M_\alpha}{I_y}\alpha + \frac{M_{\dot{\alpha}}}{I_y}\dot{\alpha} + \frac{M_q}{I_y}q + \frac{M_{p\beta}}{I_y}p\beta$$

$$\dot{r} - \left\{\frac{I_x - I_y}{I_z} + \frac{N_{pq}}{I_z}\right\}pq = \frac{N_\beta}{I_z}\beta + \frac{N_p}{I_z}p + \frac{N_r}{I_z}r + \frac{N_{p\alpha}}{I_z}p\alpha$$

Standard notation is used in these equations.

The above equations were solved on an analog computer,[14] utilizing for the main part slender body derivatives with the exception of $C_{Z\alpha^2}$ and C_{n_r} which were obtained for a wing oscillating in a viscous fluid. The comparison of the experimental and theoretical trajectories is given in Fig. 7. It is seen that the equations of motion selected provide a reasonable prediction of the motion. The lack of exact correlation may be due to either insufficient terms in the equation or due to poor theoretical stability derivatives. Attempts were made to improve the correlation by two methods:

(*a*) varying of individual derivatives one at a time by as much as 100 percent, and

(*b*) systematically varying several derivatives at the same time

The only improvement to the correlation was the introduction of incidence and compressibility effects as described by Ribner and Malvestuto.[15]

One of the main features of the analog study is a realization of the significance of the aerodynamic and inertial cross-coupling terms on the model's motion. The three aerodynamic cross-coupling terms are $C_{1_{\alpha\beta}}$, $C_{m_{p\beta}}$, $C_{n_{p\alpha}}$. A comparison is given in Fig. 8*a* of the trajectory predicted by the five degrees of freedom equation and the trajectory with the above derivatives ignored. The major term is $C_{1_{\alpha\beta}}$ which for low-aspect-ratio wings has reached large values and has dominated the damping in roll C_{1_p}. From Figure 8*a* it is seen that not only does the rolling moment reduce to the linear-motion exponential decay, but pitch and sideslip are substantially different from the full five degree of freedom motion. Ignoring the $C_{m_{p\beta}}$ or $C_{n_{p\alpha}}$ terms does not produce significant alteration in the motion in a short flight yet in longer flights changes would be significant.

The inertial cross-coupling terms

$$\frac{I_y - I_z}{I_x}\,qr, \qquad \frac{I_z - I_x}{I_y}\,pr, \qquad \frac{I_x - I_y}{I_z}\,pq$$

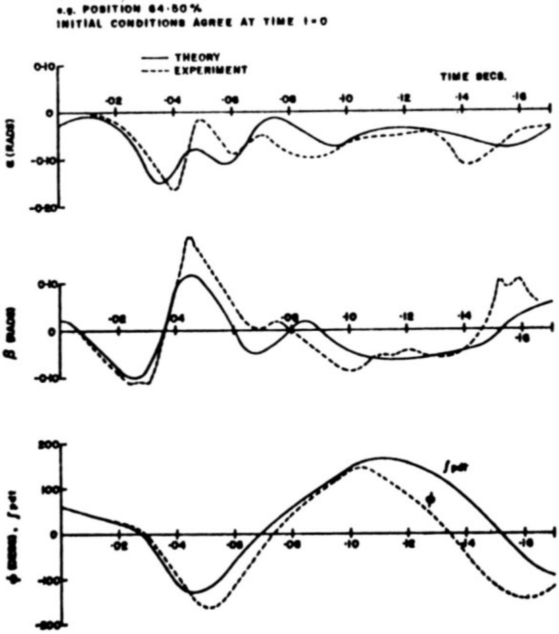

Fig. 7. Comparison of theory with experiment (pitch, sideslip and angle of bank).

due to the interaction of aerodynamic and gyroscopic forces incurred by disperse mass distribution have a marked effect on the motion and contribute largely to the non-linearity of the motion. In Figure 8*b* the motion predicted by the full set of equations is compared with that predicted by the omission of the inertial cross-coupling terms. For the slender designs considered here, the differences in inertia in pitch and roll increase rapidly over their non-slender counterparts, thus the resultant multiple of $(I_x - I_y)$ and pq for large roll rates can reach dominant proportions as is shown in Fig. 8*b*.

By applying some simplifying assumptions to the general equations, a characteristic equation may be obtained, the roots of which indicate four modes. The general equations were reduced by assuming that the non-linearities are not too violent, that vortex lift is negligible (i.e., incidence fluctuations restricted to 2–3 degrees), and that a constant roll rate is observed. It was found that the pitch incidence consisted of two oscillations: a highly damped short-period oscillation and a lightly damped long-period oscillation which appears as a constant state over the short flights in the ballistic range. The values of pitch and sideslip for infinite time were found to be dependent on purely aerodynamic effects. For uncontrolled flights it can be shown that pitch and sideslip have finite values which depend on the roll rate. Thus the motion does not tend to zero as $t \to \infty$ but slowly oscillates so that in actual flight auto controls would be required.

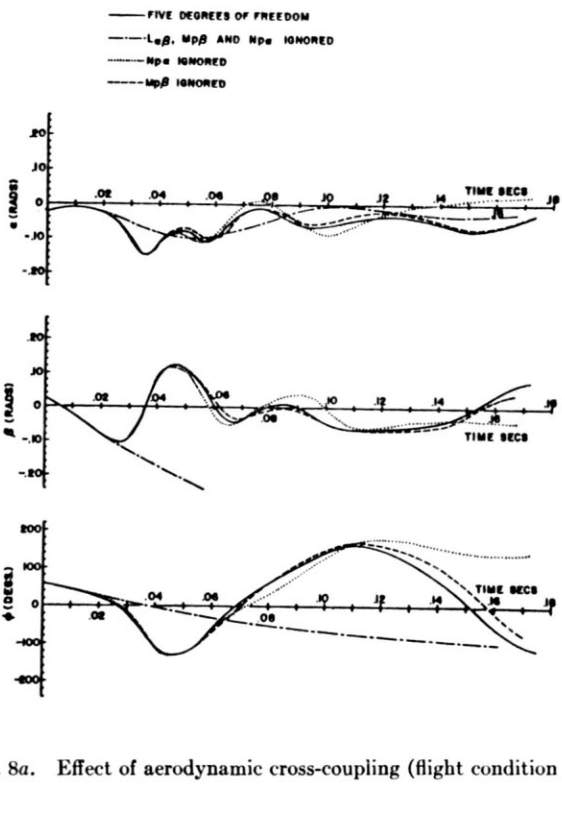

Fig. 8*a*. Effect of aerodynamic cross-coupling (flight condition *A*).

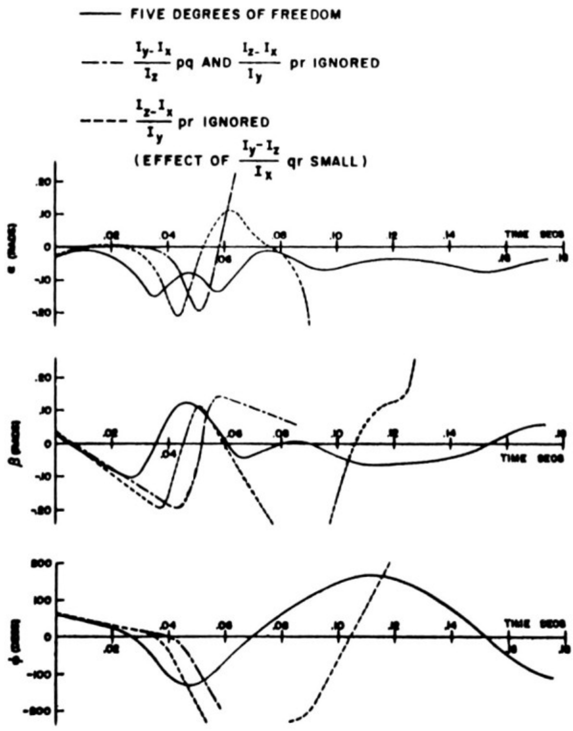

Fig. 8*b*. Effect of inertia cross-coupling (flight condition *A*).

ANALYSIS METHODS

The method for extracting stability data from the measured oscillatory history for bodies possessing trigonal or greater symmetry including small non-linearities in the aerodynamic moments has been previously described. The flat wings do not possess the necessary symmetry and they also exhibit large nonlinearities and cross-coupling. Thus the problem is to devise a method by means of which the five unknown stability derivatives which are influencing the motion may be extracted from the range trajectory data. The accepted connotations of period and frequency of motion and logarithmic decrement of the oscillations cease to have a significance for the motion exhibited by the flat plate wings. Thus no simple reduction procedure can be used to obtain either a damping factor or the static stability parameter.

No satisfactory method has been obtained to extract all of the stability derivatives from the wing's motion; however, the two methods investigated at CARDE, a modified Fourier transform method and an equivalent differences method will be described briefly in this report. Fuller details of the methods are given in Ref. 16. The main problem that has been experienced in both analysis methods involves insufficient accuracy of the raw data.

The modified Fourier transform method entails transforming the differential equations of motion into linear integral equations which are then solved by a least-squares technique. This method, suggested by Shinbrot,[17] consists of multiplying the equations by some chosen smooth analytical function $[y_\nu(t)]$, integrating over a specified interval of range data and solving the resultant set of linear equations by the method of least squares. The equation describing the pitching motion may be written as

$$\ddot{\alpha} + a\dot{\alpha} + c\alpha + e\alpha\dot{\alpha} + f\alpha^2 + j\dot{p}\beta + mp\beta = \frac{I_z - I_x}{I_y} p^2\alpha - \left(1 + \frac{I_z - I_x}{I_y}\right) p\dot{\beta} \tag{10}$$

By multiplying through by $y_\nu(t)$ and integrating over the flight time in the range we obtain

$$aA_{\nu 1} + cA_{\nu 2} + eA_{\nu 3} + fA_{\nu 4} + jA_{\nu 5} + mA_{\nu 6} = B_{\nu 1} + B_{\nu 2} + B_{\nu 3} \tag{11}$$

where $A_{\nu n}$ and $B_{\nu n}$ are functions of the initial conditions and integral functions of the range data. The aim of the transformation of the differential equations to integral equations is to avoid the usual errors incurred in the conventional difference equation techniques where double differentiation of experimental data is required.

The function suggested by Shinbrot was $y_\nu(t) = \sin^2 \omega_\nu(t)$ were $\omega_\nu = \nu\pi/T$, $\nu = 1, 2, 3 \ldots N$ where $N \geqslant$ number of unknowns and $0 < t < T =$ flight time. This function allows

$$y_\nu(T) = y_\nu(0) = \dot{y}_\nu(T) = \dot{y}_\nu(0) = 0$$

which thus removes from the above equation any dependence on end conditions. The above equation may be solved by a method of least squares for $a, c \ldots m$ which are functions of the stability parameters. This results in a set of six linear equations in six unknowns which can be solved by some routine method. This set of equations then involves the measured range data, the attitude angles as a function of the flight path. Although this method as applied to the delta wing study was unsatisfactory, the method was not exhausted and the particular failures will be briefly given here.

The solution of the linear equations involves the solution of a matrix, each element of which is a summation over the range $1 < \nu < N$ which is in turn an integral over the range $0 < t < T$. To obtain a reliable estimate of the matrix the functions

(i) $d/dt(A_{\nu j})$ versus t (or X)

(ii) $A_{\nu j}$ versus ν (number of unknowns)

must be well defined and accurately determined. Condition (i) implies that a large number of observations are required within the range length (total flight time). The reason for this comes from the fact that the two harmonic functions $y_\nu(t)$ and $\alpha(t)$ are multiplied together to produce a nonlinear curve with a large number of zeros. The choice of the method function $y_\nu(t) = \sin^2\omega_\nu t$ has the unfortunate characteristic of producing zeros in the range $0 < t < T$. A function such as $y_\nu(t) = \sin \omega_\nu t$ has not been tried although it would reduce by half the number of zeros. Other functions such as a polynomial or exponential tend to magnify the experimental errors and would produce a singularity in the matrix required for the solution of the linear equation. The Fourier transform method above was not further pursued in our studies.

The method of equivalent differences was also investigated, although here the accuracy of the experimental data must be very high since it entails the double differentiation of the experimental data. The method involves the replacement of the differentials by their equivalent differences. In order to compute the derivatives of the range data a five point polynomial is used with the derivative being evaluated at the mid point. The resultant set of linear equations utilizing the measured attitude data may be solved by a standard method.

EXPERIMENTAL RESULTS

The wings were launched into the CARDE Aeroballistics Range with the attitude data obtained from model signatures on a series of light weight cards mounted perpendicular to the flight path. Velocity decay was also measured. The models were launched with zero roll rate; the initial model disturbance is due to model-sabot interference at separation and in some experiments due to an initial pitch incidence of the model in the sabot. These tests were conducted at low supersonic velocities up to about Mach 2 under ambient atmospheric conditions.

The typical α, β, ϕ motion of a delta wing has been shown in Fig. 7 under the discussion of the analog simulation results. The above discussion of the analysis techniques has not yielded a satisfactory method to handle all of the unknowns, however, the rolling motion equation has been solved to provide C_{1_p} and $C_{1_{\alpha\beta}}$ by the equivalent difference method.[18] The roll equation is

$$\ddot{\phi} = \ddot{\phi}\ (0) \exp\ (K_1 C_{1_p})t - K_2 C_{1_{\alpha\beta}} \exp\ (K_1 C_{1_p})t \int_0^T \exp\ (K_1 C_{1_p})t\ \alpha\beta\ dt \tag{12}$$

where

$$K_1 = \frac{\rho U S l^2}{2 I_x} \qquad K_2 = \frac{\rho U^2 S l}{2 I_x}$$

The experimental values of C_{1_p} and $C_{1_{\alpha\beta}}$ are given in Figs. 9 and 10. The theoretical results of Ribner and Malvestuto consist of a modification to the slender body theory to account for the proximity of the Mach cone to the wing leading edge. The leading-edge suction forces are considered by the viscous effect of the leading edge vortex flow and the effect of the spanwise variation in Mach number due to yawing have not been included. It is seen from the experiment that a considerable discrepancy exists from the theoretical predictions.

The experimental values of C_{L_α} obtained from the range trials are compared in Fig. 11 with some wind tunnel measurements and flat-plate wing theory. This data is obtained from selected trials in which the oscillatory motion of the wings approaches an equilibrium value. For this case the equations of motion are simplified by setting time derivatives equal to zero and by ignoring small forces and moments due to sideslip. An expression may be derived relating C_{m_α} to the square of the equilibrium roll rate which in turn is related to C_{L_α} and the wing center of gravity.

It should be noted that the buildup in roll of the wings up to 200 radians per sec was incurred by aerodynamic effects and that frequently the wings accelerated in roll through the critical roll rates for resonance and divergence. The acceleration in roll in the case of the wings was too rapid for divergence to occur.

APPLICATION OF ACCELEROMETERS

It has been shown that an inherent weakness in the ballistic-range technique involves the accuracy of the attitude and position data. The accuracy of the raw data is particularly important when intercoupling of the longitudinal and

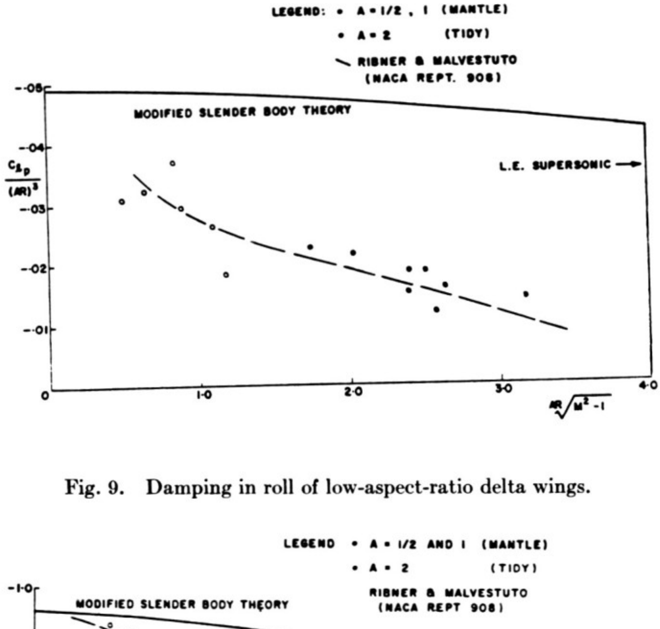

Fig. 9. Damping in roll of low-aspect-ratio delta wings.

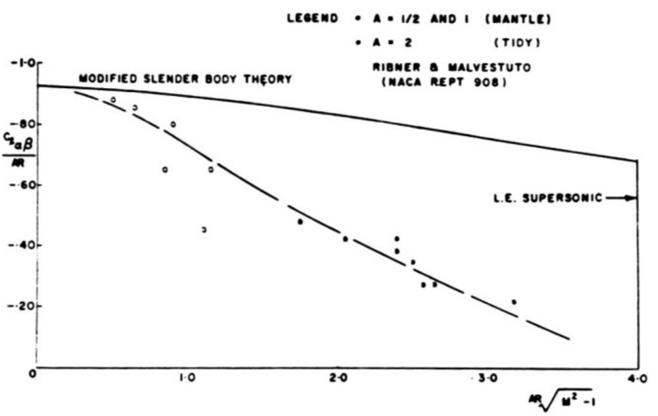

Fig. 10. Nonlinear sideslip of low-aspect-ratio delta wings.

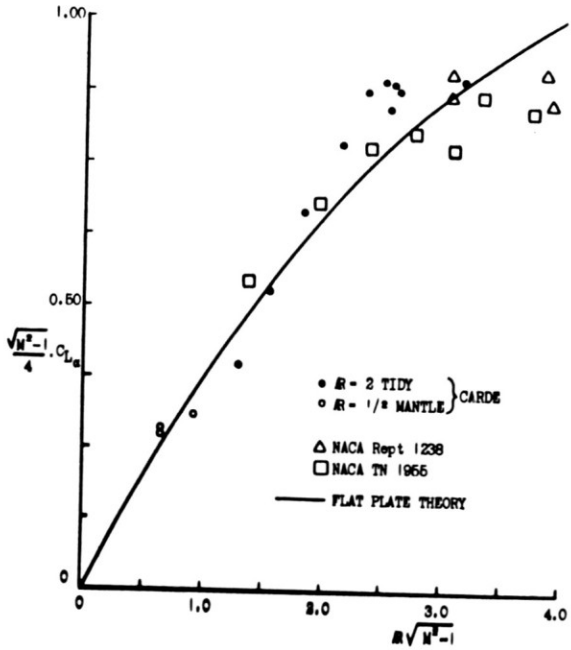

Fig. 11. Lift-curve slop of low-aspect-ratio delta wings.

lateral motion exists. Since discrete measurements are taken at fixed intervals in the range, they cannot necessarily record the peaks of the oscillatroy motion; thus a continuous record eliminates a great source of error. In particular, the continuous measurement of in-flight accelerations coupled with discrete attitude and position coordinates provides a reliable record of raw data for input into the equations of motion. Accuracy is increased due to the continuous record of motion as well as by elimination of the double differentiation procedure.

The accelerations at any point in a body may be written in terms of body axes as

$$
\begin{aligned}
l &= \frac{U}{g}\,(\dot{\beta} - \alpha p + r) + (pq + r)\,\frac{x}{g} + \sum C_i' \\
a &= \frac{U}{g}\left(\frac{\dot{u}}{U} - \beta r + \alpha q\right) - (q^2 + r^2)\,\frac{x}{g} + \sum C_i'' \qquad (13) \\
n &= \frac{U}{g}\,(\dot{\alpha} + \beta p - q) + (pr - \dot{q})\,\frac{x}{g} + \sum C_i'''
\end{aligned}
$$

where the correction terms $\sum C_i$ account for the physical size of the sensitive element, the errors in physical location of the mass center of the accelerometer and any cross-axis sensitivity in the element.

To the present time, measurements made at CARDE of model accelerations have not been applied to any cases of nonlinear motion but have been restricted to axisymmetric configurations in order to gain some familiarity with the difficulties of manufacture, calibration, and analysis.

CONCLUDING REMARKS

The application of the ballistic-range technique to the study of the dynamics of vehicles has been described with particular emphasis on the inherent problems of lifting vehicles under uncontrolled flight. It has been shown that the analysis of free-flight motion of scaled models can provide much data necessary for the computation of a vehicle's trajectory, control, and maneuverability. However, much more data is available from the model's flight than the current analyses are capable of extracting, even with the relatively simple measurements described. Coupling this information with data from telemetering, radar, radiation, and other methods provides a considerable mass of interdependent data from which a good understanding of the flow phenomena may be obtained.

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Discussor: F. C. Haus, Universities of Ghent and Liege

I was interested by Fig. 8, which showed the difference of behavior when $L_{\alpha\beta}$ is considered and when it is ignored. I was myself making analog computations about coupling during roll and I found the derivative $L_{\alpha\beta}$ was among the most important factors.

I came to the conclusion that the approximate theories of coupling, which ignore the derivative $L_{\alpha\beta}$, are not accurate enough to give a good explanation of airplane motion and wonder if this is also the opinion of the authors.

Author's reply to discussion:

Although I am not fully aware of the details of the approximate theories of coupling to which you refer, I would agree that any theory for predicting the motion of airplanes which ignores the dihedral or sweepback effect and its variation with angle of attack, will not adequately describe the motion.